Understanding Geometric Pattern and its Geometry Part 11 – Using Anatolian Seljuk Architecture as a Source of Inspiration for Students' Projects in Mathematics

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Abstract

In recent decades, with the continuous changes in the mathematics curriculum, many geometry-related topics were neglected or removed from our classroom activities. Thus, some teachers pursue these topics as independent students' projects. There is also a growing interest in various courses and workshops dealing with practical geometric pattern design.

The Author has taught courses and conducted workshops on geometry and pattern design at the Istanbul Design Center for many years. Participants in these events were high school and university students, artists, and architects.

In this paper[2], we will discuss selected concepts of geometry used by Seljuk architects in designing the famous Seljuk geometric mosaics and stone carvings. We will show how these geometric concepts were used in selected examples from mosques, tombs, hans, and madrasas. We will also demonstrate using geometry software, e.g., Geometer's Sketchpad or GeoGebra, for constructing geometric patterns.

Introduction

In most Muslim communities, we can find mosques and other buildings adorned[3] with geometric, floral, or mixed decorations. These decorations follow strict geometric rules, and most of them can be modeled using traditional school geometry tools – ruler and compasses, or modern technology – computer software like GeoGebra or Geometer's Sketchpad. In the first case, we are often stacked with inaccuracies of drawing by hand. In the second case, the drawings are precise, accurate, and can be done much faster. This way, students have more time to investigate the properties of objects and complex geometric concepts. In this paper, all drawings were created with Geometer's Sketchpad, now a free tool for teaching and exploring geometry. For more information on using GSP, check [7] .

In this paper, we deal with geometric adornments from Anatolian Seljuk architecture. In a separate paper, Ottoman geometric adornments will be discussed.

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[1] *In 2018, New York Tech announced that it will no longer enroll students at its Abu Dhabi campus. A few years later the Abu Dhabi campus was closed.*

[2] *The original paper was presented during the 6th International Symposium of Turkish Computer and Mathematics Education (TURCOMAT-6) organized by the Mathematics Education Association and the Turkish Journal of Mathematics Education, hosted by Başkent University, Faculty of Education and was held in Kızılcahamam on 28-30 October 2023. Due to the size of the material the author decided to split his talk in two parts related to (1) Seljuk architecture and (2) Ottoman architecture. Still this paper is an expanded version of my TURCOMAT-6 talk.*

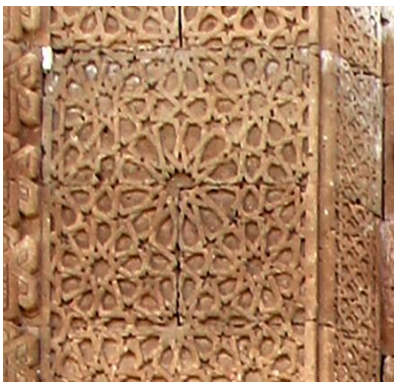
[3] *Most of architects insist on using term adornment instead of a geometric pattern. It makes a sense. Geometric pattern it is a pattern that we draw on a paper more or less accurate. In fact, this is linear often simplified representation of an adornment. Adornment is a material representation of a geometric pattern or a floral design. Here the used the term material matters. Patterns painted on a wall look completely different than the same patterns carved in stone or made in wood.*

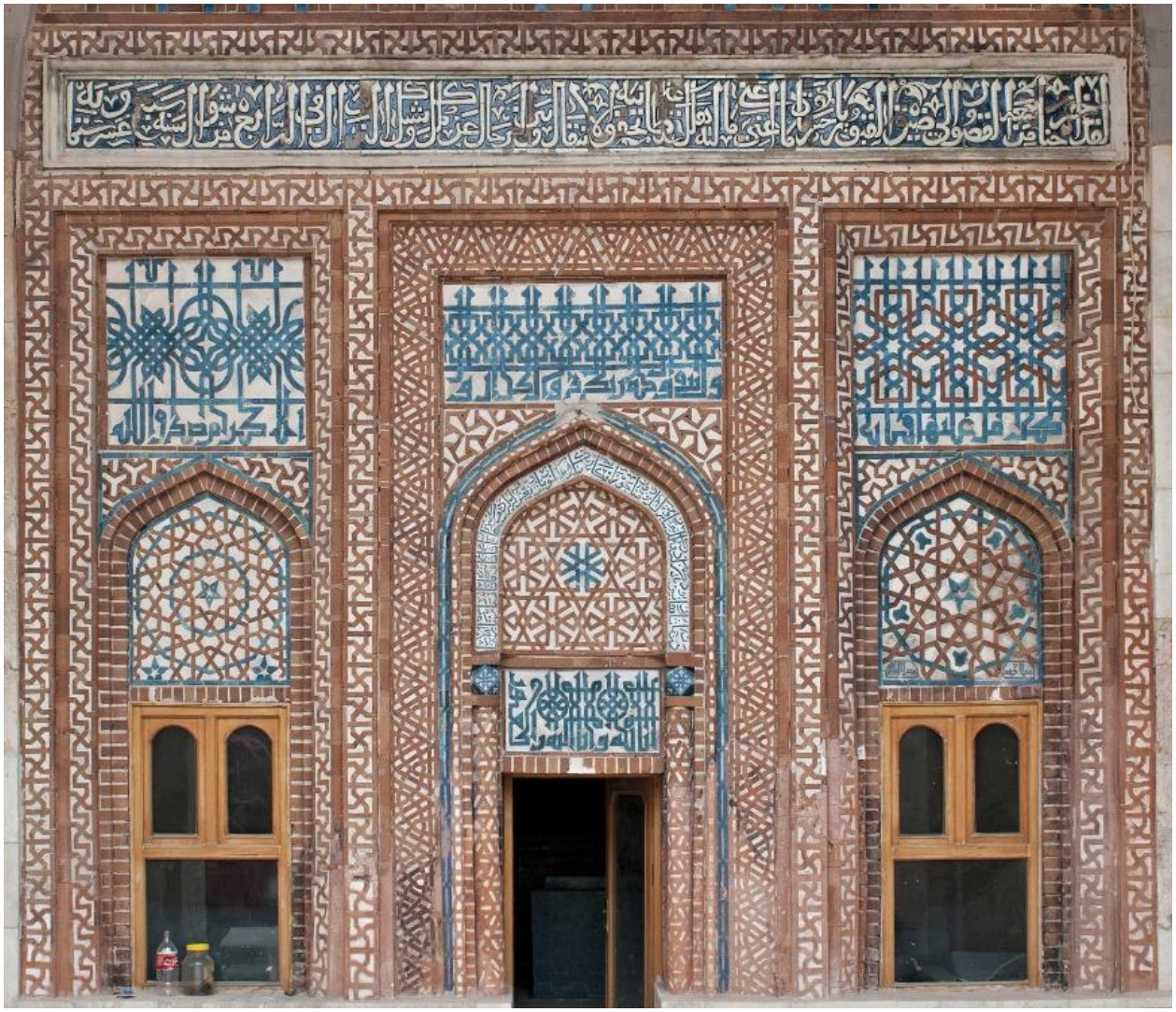
Adornment of the wall in Ağzıkara Han

Ağzıkara Han is a historic Seljuk-era caravanserai in Turkey. It is located in the Ağzıkarahan village in the province of Aksaray.

Many Anatolian Seljuk architectural structures have typical massive walls with similar decoration. Here, the complete adornment was carved in large stone slabs. Usually, there are a few long vertical freezes. Often, they are fragments of larger plane patterns.

Below – selected fragments of the adornment

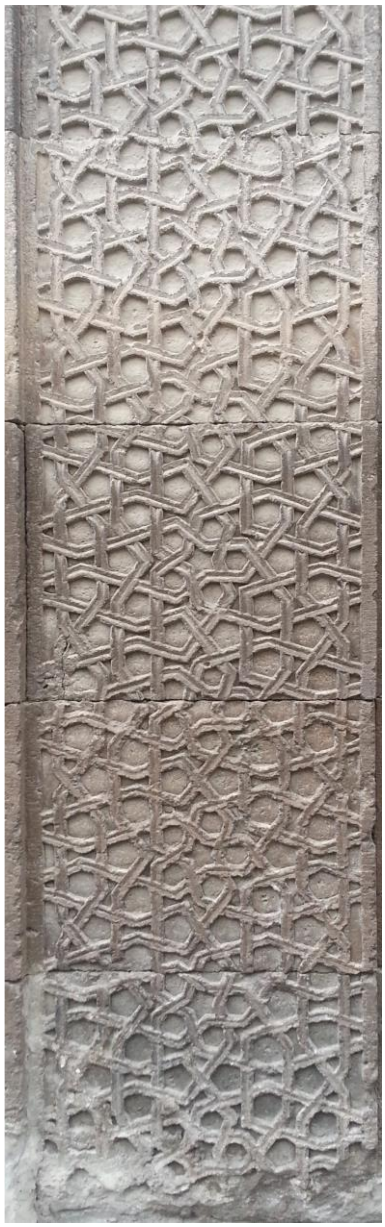


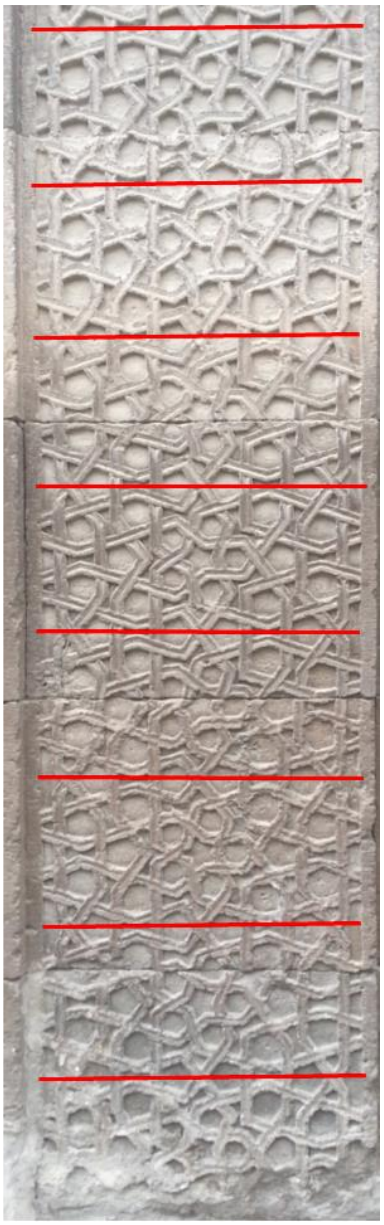


Tomb of Izz al-Din Kayka'us I, in the Şifaiye Medrese (1217–1218), Sivas

Such adornments can be analyzed and reconstructed in two or more steps. We will start with step 1.

STEP 1 – Choose the pattern template





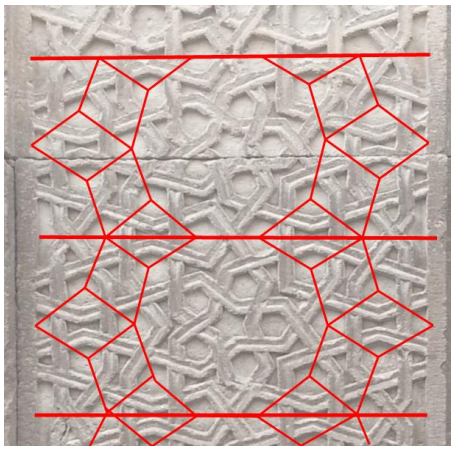
Fragment of a wall from Hunad Hatun complex in Kayseri, Anatolia, Turkey

The photo to the left shows a typical Seljuk adornment from Anatolia. It is a maze of segments carved in blocks of stone. The carving is inaccurate and often destroyed by weather. For an average viewer, the structure of the pattern is almost impossible to decipher. Thus, we need to split the whole pattern into repeating fragments.

The right illustration shows how the whole design can be divided into rectangles. Each rectangle is a mirror copy of the rectangle above and below. Thus, one has to draw one of these rectangles and repeat it a few times up and down. This way, we can get a replica of the whole adornment.

From this example, we can conclude that many geometric adornments can be reconstructed from rectangular or often square designs by repeating them with reflections about the edges of such rectangles.

NOTE – *in mathematics, we often consider the so-called symmetry group of a pattern. The smallest fragment of the pattern necessary to reconstruct it is called a fundamental region of the pattern.*



Analysis of the fundamental region of the pattern from Kayseri

This illustration requires a bit more experience. Many geometric patterns can be analyzed by wrapping some shapes into convex and symmetric polygons. Some of these polygons can also be regular polygons. However, most of them are only symmetric with one or more symmetry lines. In this illustration, we have regular decagons, rhombi with equal edges, and halves of decagonal octagons (on the left and right edges). We will discuss them in detail later in this paper.

The origins of the polygonal approach can be found in the Topkapi Scroll, a Timurid dynasty pattern scroll in the collection of the Topkapi Palace Museum.

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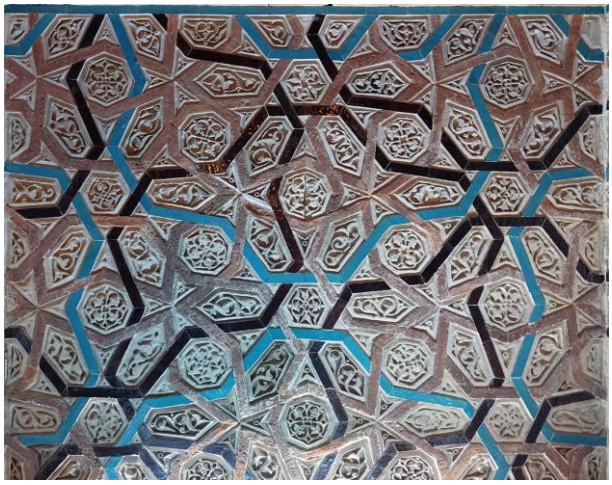
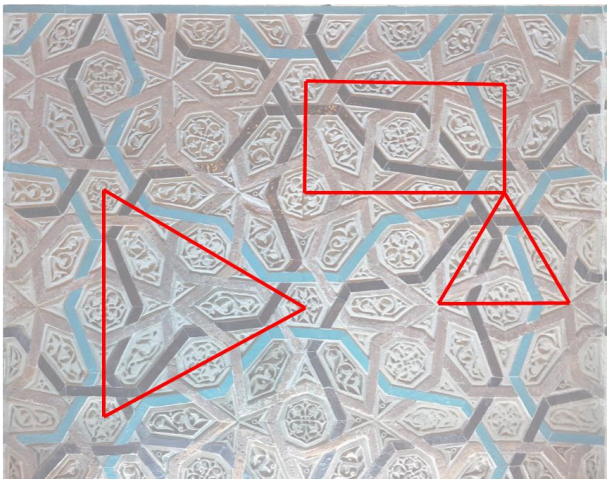




Adornment from Ahî Şerafeddin Mosque in Ankara

The wall mosaic presented here can be examined and reconstructed in a few different ways. Below, we present some of them.

1. The most frequent approach is to construct the rectangular part of the pattern, the so-called template, and then use reflections about its edges to produce the whole design. This is the traditional approach based on the methods shown in the Topkapi scroll.
2. We can use one of the two triangles shown in the illustration as a template. This is a modern approach based on symmetry groups. One can notice that the pattern has three types of points:

   Centers of hexagons. Here, the pattern can be rotated about them six times, each time 60 degrees.

   Centers of octagons where the pattern can be rotated twice 180 degrees.

   Center of the large triangle where the pattern can be rotated three times, 120 degrees each time.

   Thus, after reconstructing one of these two triangles, we obtain a fragment that should be rotated about its vertices.

   Like in the previous example, we could wrap shapes of this pattern into symmetric polygons, but this may not be necessary.

IMPORTANT NOTE

> *In both examples, we ignored the fact that we deal with interlace designs. In a non-interlace design, we use reflections of the template about its edges. In an interlace design, we need larger rectangular templates and translate them along their edges or rotate them about some points on their edges. The interlaced geometric adornments will be discussed later in detail.*

The two examples presented here bring us to various types of symmetries – reflections, rotations, etc., polygons regular and symmetric. Sometimes, we will deal with semiregular polygons, i.e., polygons with all edges equal and different angles. Note also each time while we discuss polygons related to a pattern, we deal with a geometry of a particular type. In the Hunad Hatun example, we dealt with shapes originating from the regular decagon. In the Ahî Şerafeddin example, we deal with equilateral triangles and hexagons. Thus, we refer to the first example as decagonal and the second as hexagonal.

In the following sections, we provide more details about each type of geometry.

STEP 2 – find out the type of geometry

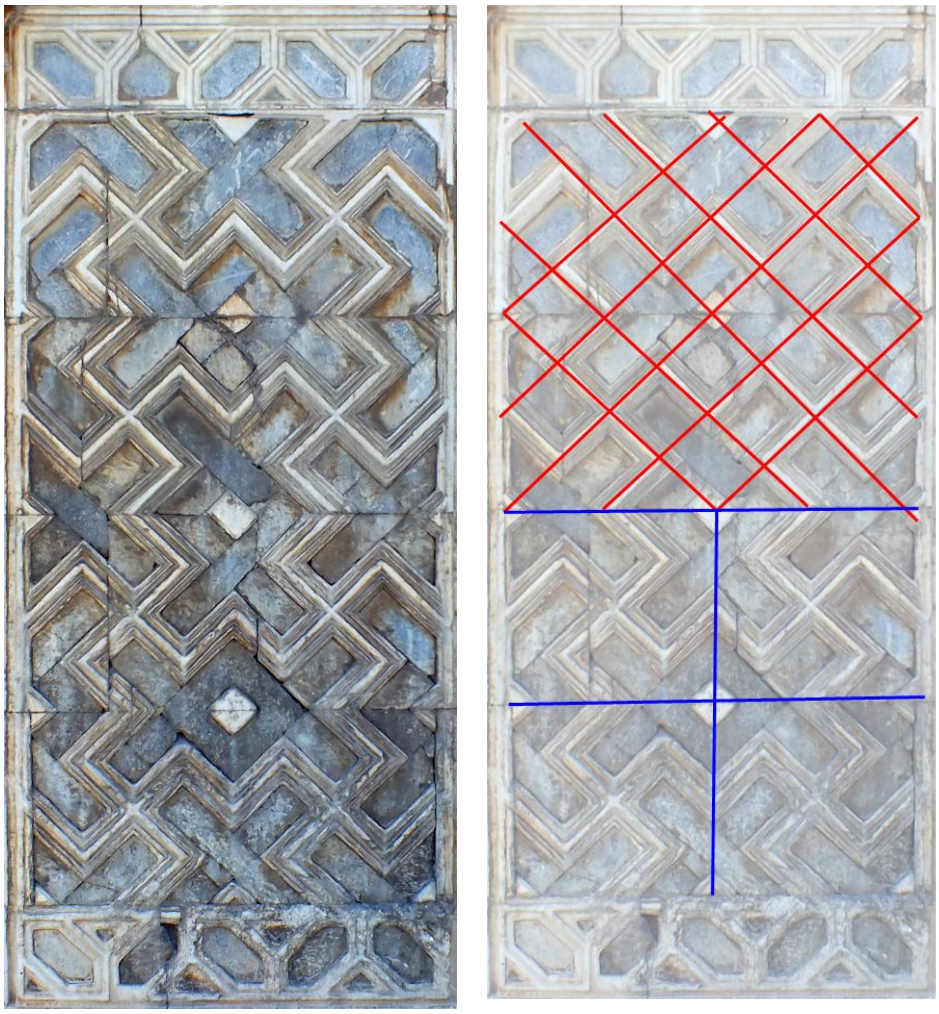
For most geometric adornments, we can easily decide what type of geometry they represent. The first example was built in a decagonal geometry. The second one is more difficult. Although it can be created from triangular or hexagonal[4] templates, but it contains regular octagons and uses angles from both regular octagon and regular hexagon. The most important is to notice that regular octagons are only a specific feature of this pattern. We do not have points on the pattern where at least a part of it can be

[4] *An equilateral triangular template can be rotated six times about one of its vertices forming this way a new hexagonal template. Often it is very convenient approach.*

rotated eight times around them. At the same time, we have points where the whole pattern can be rotated six times around them. We have there also some hexagonal stars and rosettes. Thus, it is convenient to consider it as a hexagonal design.

In further examples, we will discuss geometric patterns using the geometry of a square, a regular hexagon, a regular octagon, a regular decagon, etc. Patterns using geometries of multiple regular polygons, i.e., patterns using the geometry of a regular dodecagon and octagon, are not discussed here.

Anatolian Seljuk patterns using the geometry of a square



Geometric adornment from the wall in Karatay Madrasa in Konya

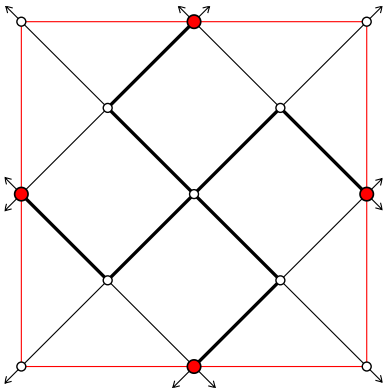
In the photo, we have a design using right angles with thick frames. If we ignore the width of frames, we get a simplified representation of this pattern.

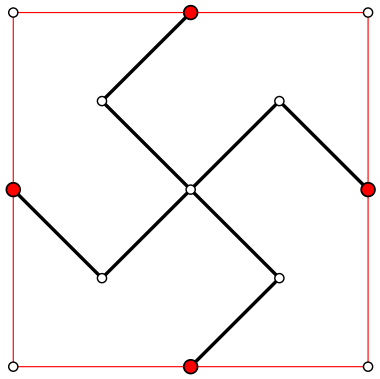
The two methods of redesigning this pattern are shown in the right picture. We can treat it as a pattern made on a square grid (red lines) or a pattern made using a square template (blue lines).

The first method is time-consuming, and it can lead to many errors. But it can be used to draw several different patterns.

The second method is more convenient and much simpler. We will follow it.

The design obtained from this template will be a linear representation of the adornment. It is a great device to develop an overview of the pattern. In reality, we must create it from a given material, i.e., marble slabs or wood pieces. Thus, we must decide about the thickness of linear elements, their profiles, and their interactions. Their profiles can be flat, round, or ridged. They can be connected to each other in a few ways or interlaced. Thus, we can investigate these options and see differences in their constructions.

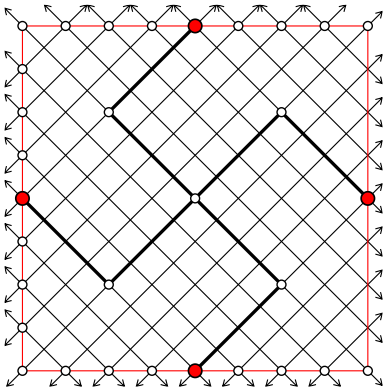


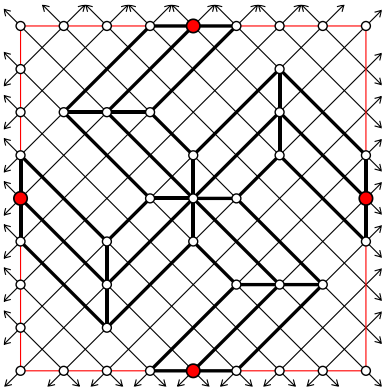


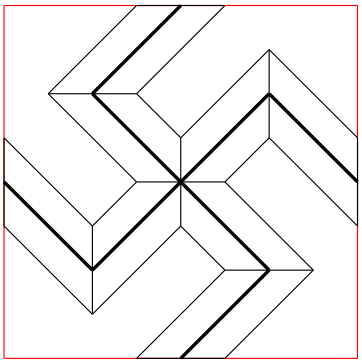
Left – construction of a simplified template.

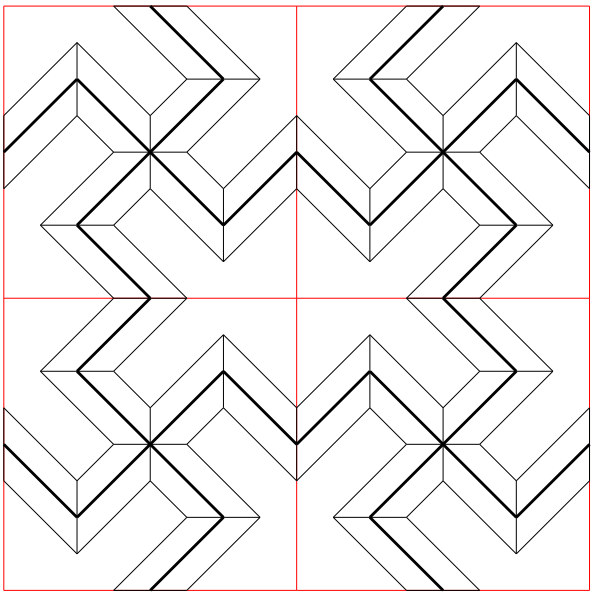
Right – final simplified template

The template obtained in this example can be used to create the complete pattern from Karatay Madrasa. We need four copies of it and four mirrored copies.







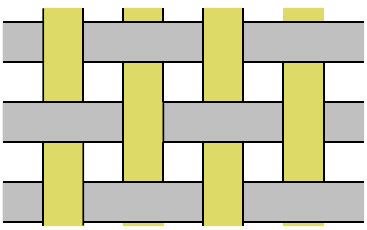


Above – three steps for creating a realistic template. We divided each edge of the square into eight equal parts. Then, we draw the square grid (thin lines). Finally, we enlarge the width of each edge of the pattern. The divisions of each triangle edge can be much denser, so we could mimic the ridged profile of straight elements.

Left – half of the pattern created from two copies of the template and two mirrored copies of it.

We could use this large square design as a new template. Then, we do not need any more mirrored elements. We could assemble a large pattern in the same way a craftsman covers our floors or bathroom walls by placing each tile next to another one.

Let us assume we want to draw an interlaced model of the same pattern. We can start from one of the above drawings and adapt it in such a way that the belts of the adornment will interlace. What does this mean?



A typical interlace pattern

Elements of this design go over and under other elements. They form sequences of alternating intersections: ... 1,0,1,0,1,0,1,..., where 1 means that the element is over another element and 0 means that the element is under another element. Thus, sequences like these 1,0,0,1,... or 0,1,1,0... are invalid for an interlace design.

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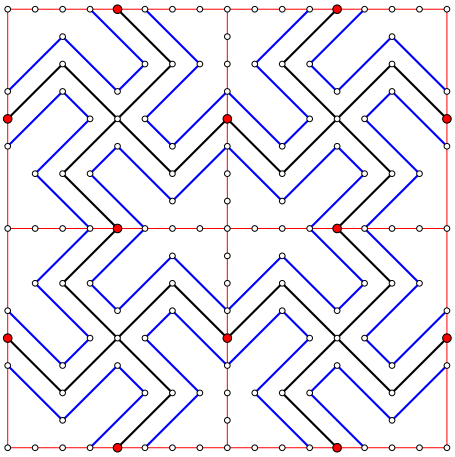
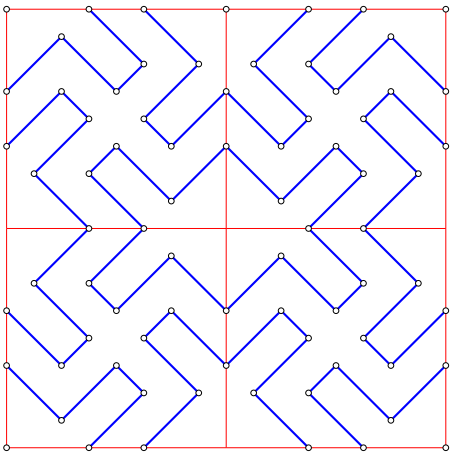
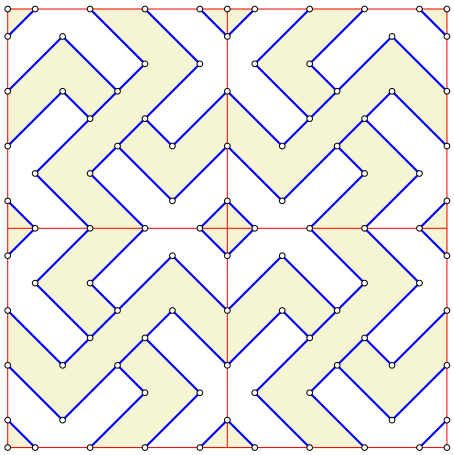


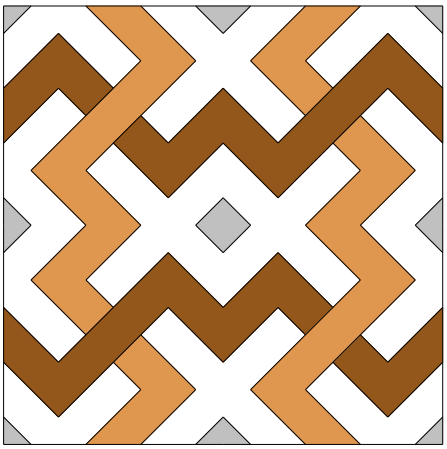


Steps for creating an interlace version of the pattern from Karatay Madrasah

The left drawing is essentially the same as the one from the previous design.

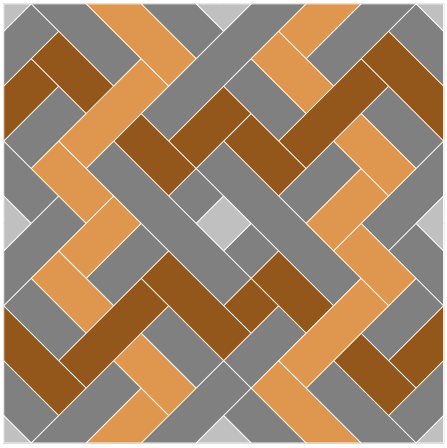
We removed all central lines (black segments) in the right drawing.

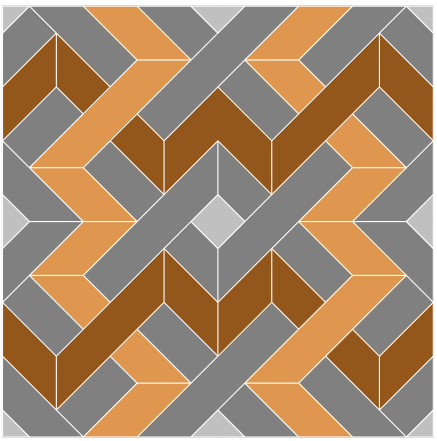




The left image shows how to modify the previous drawing to get the interlace of elements.

The right drawing shows half of the adornment from Karatay Madrasa made in an interlaced form. Note – we added gray squares in the center of each large element.





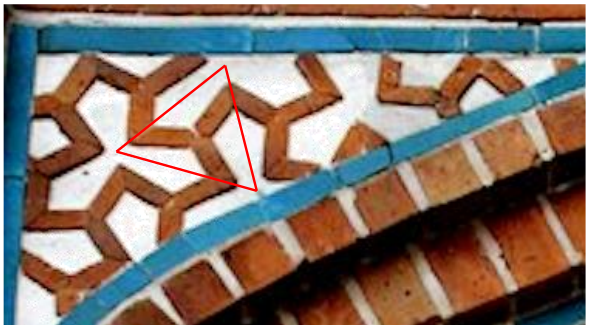
Two more realistic versions of the pattern from Karatay Madrasa

The approach in the left drawing is more convenient for a stone master. Such a design can be easily created using clay bricks. The blue color is the background for both designs.

Anatolian Seljuk patterns using hexagonal geometry

In Anatolian Seljuk architecture, we often find two types of geometric adornments that can be classified as hexagonal. In the first group are designs created with so-called isomorphic grids. In the second group are designs made with triangular templates and often complex geometric constructions of the detailed pattern. We can also find designs using some hexagonal tessellations. But these designs are rather rare.

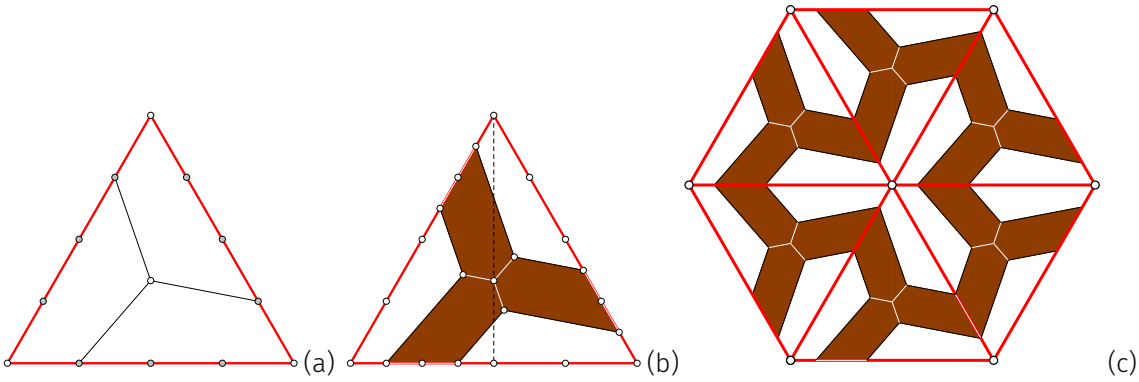
All three cases will be briefly demonstrated here.

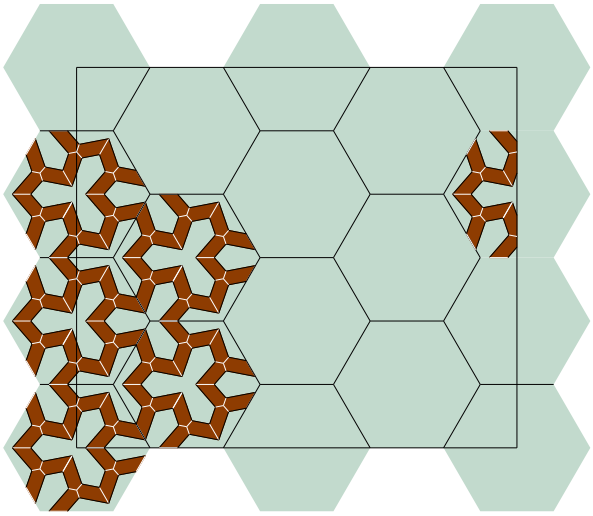


Tomb of Izz al-Din Kayka'us I, in Sivas

In the photo, we have the pattern from above the main entrance. It is a small fragment of a large adornment of the front wall.

On the same wall, we can find a few other interesting patterns (check the third page of this paper).





Construction of the pattern from Sivas

(a) We start from an equilateral triangle, dividing its edges into four equal parts and connecting the first-left point with the center of the triangle.

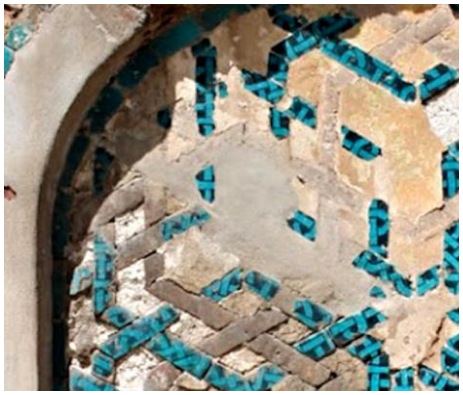
(b) One of many ways of getting a more realistic model of the pattern. The width of the pattern element can be any.

(c) A hexagon made by reflections about the edge of the drawing from (b).

Left – here, it is shown how we could fill a plane with hexagons (c).

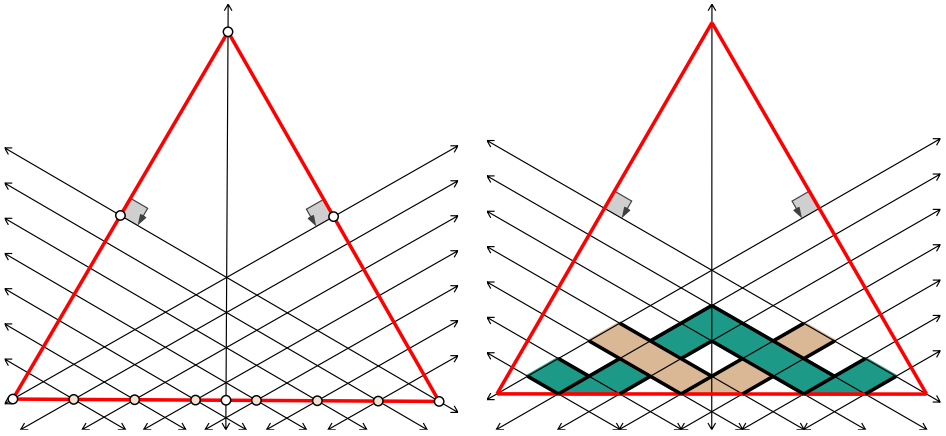
NOTE – one can divide the edge of the triangle into a different number of parts and then follow the remaining steps. This way, a slightly different pattern can be obtained.

Numerous hexagonal patterns from Anatolia can be produced from triangular or so-called isometric grids.



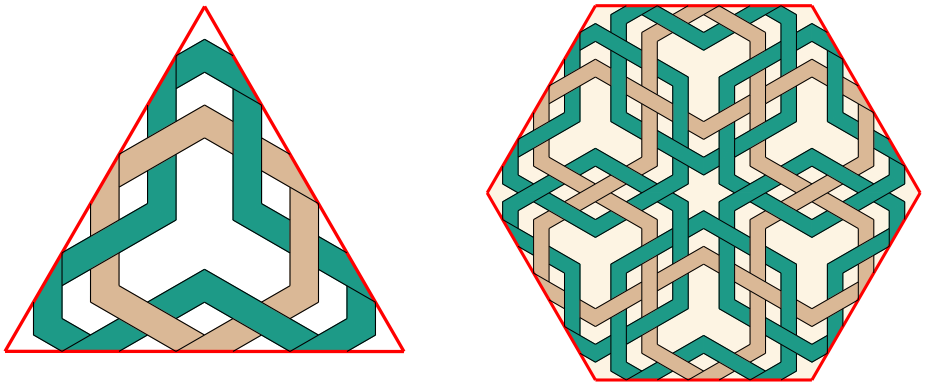
Adornment from the tomb of Ali et-Tusi in Tokat

Despite of the damage, we can decipher the construction of this pattern.



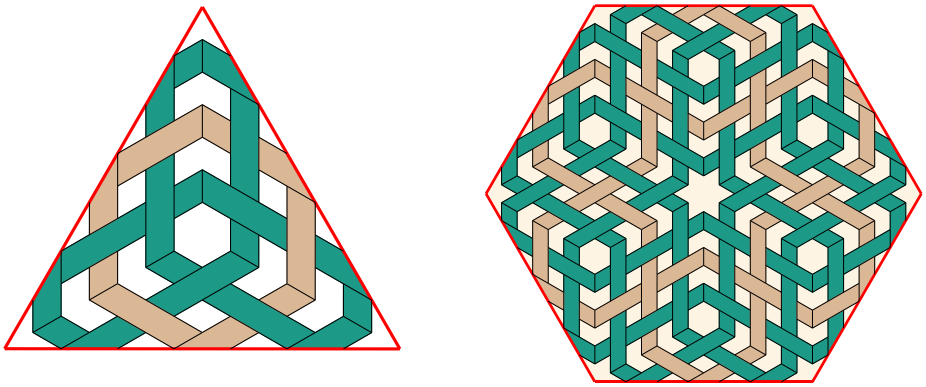
A rhombic grid for this pattern

In this construction, we assume that the width of all belts is the same. But they can be wider or narrower. Then, we will have to adjust the grid used here.



Construction of the triangular template

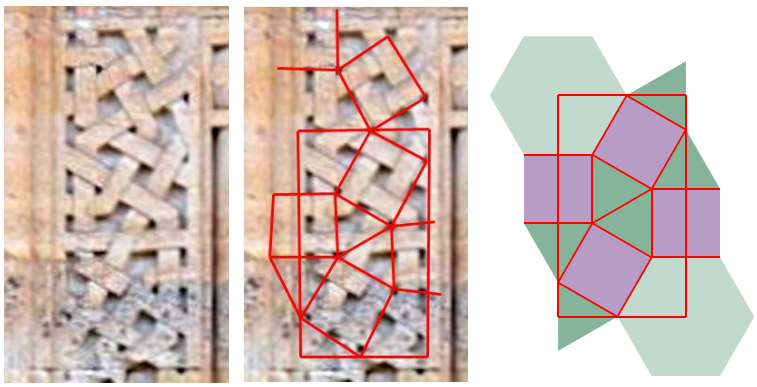
Note – the Ali et-Tusi tomb there are two slightly different patterns. Here, we model the left one. The right pattern has the center of the triangle filled with extra motifs.



The pattern from the right side of the Ali et-Tusi tomb

By skipping some segments of such a pattern, we can get variations. Also, coloring can underline some shapes and motifs.

In Anatolian architecture, we can find many other patterns made on triangular grids. Some of them are quite complex, and most of them exist only in Turkey. A few hexagonal patterns can be constructed using tessellations of polygons related to the regular hexagon.



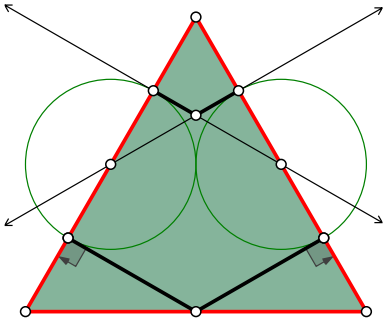
Fragment of adornment from Avanos Sari Han

The three illustrations show how one can analyze this pattern. The pattern can be designed using three polygons – an equilateral triangle, a square, and a regular hexagon. As we will see later for triangles, we will have two slightly different versions. The pattern for hexagons can also be built using a design in a triangle.

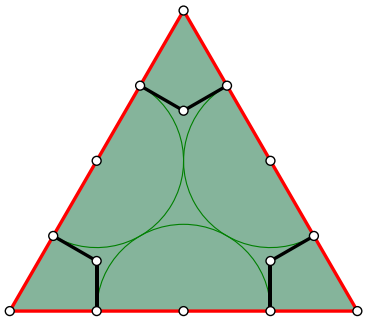
The idea presented here is very popular for designing complex patterns in decagonal and octagonal geometry. It consists of three steps:

1. Design a tessellation
2. Design a pattern for each tessellation tile so that we get smooth transitions (without any breaks) of a pattern from one tile to each one tangent to it.
3. Fill each polygon of the tessellation with the appropriate pattern.

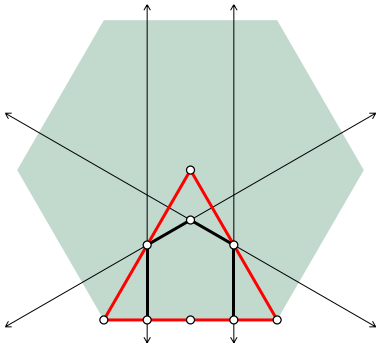
In our case, we have three different polygons, and creating a pattern for each of them is a simple task. Let us start with equilateral triangles. Note – we have here three different motifs for the triangle.



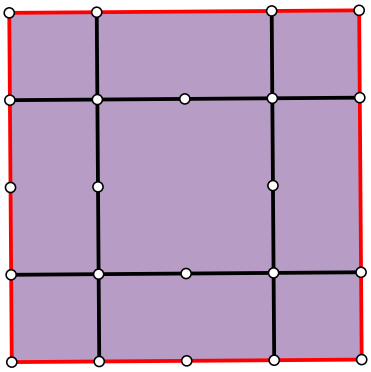
(a)



(b)

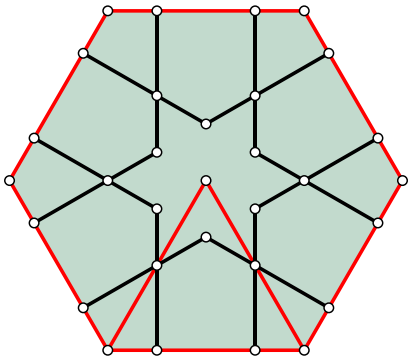


(c)

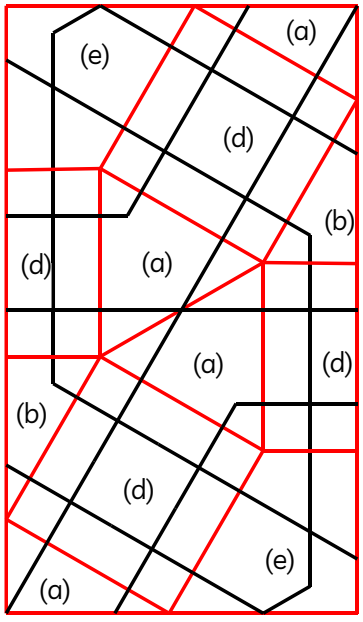


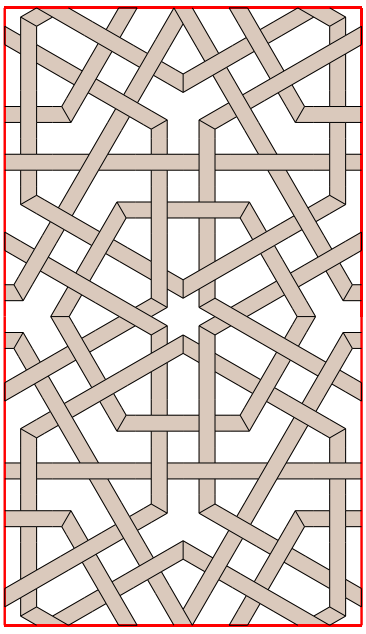
(d)

Drawing (a) shows how to start drawing the pattern for each polygon. The two segments perpendicular to the left and right edge of the triangle touch it in ¼ of its length. Therefore, dividing each triangle edge into four equal parts will be the key idea for all these drawings.



(e)





Left drawing – tessellation filled with pattern. Letters show where which fill was applied.

NOTE – there are many other ways to fill each of these polygons. This way, we can produce many pattern variants for a given tessellation.

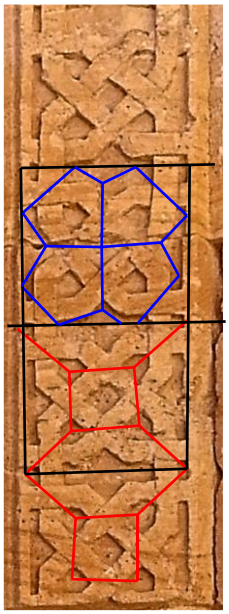
Moreover, there are hundreds of tessellations using these three polygons. Thus, using this method, one can produce hundreds of patterns, old or new.

It should also be noted that making an interlace design with this method is a simple task (right drawing).

Anatolian Seljuk patterns using octagonal geometry

In hexagonal geometry, the number of shapes that can be used to design patterns is somewhat limited and not well known. We had only a regular hexagon, square, and equilateral triangle. But there is more, and the topic is worth exploring.

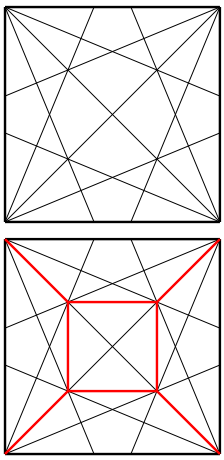
Octagonal geometry contains a significantly larger number of polygons and methods for pattern design. We will start with something very simple.



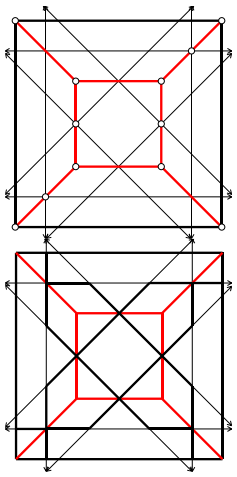
A border pattern from Saruhan, Avanos

The drawing shows a fragment of a very simple freeze. The red and blue lines show how we could approach this pattern.

We could use two types of polygons – the red or the blue. We get very simple shapes in each case, and a tessellation for each is easy to construct.

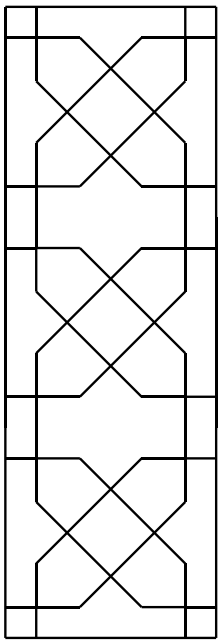


Construction of the tessellation

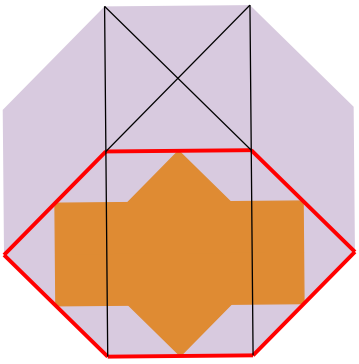


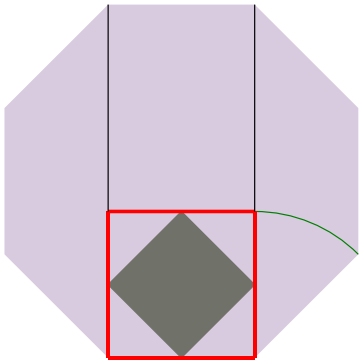
Construction of the template

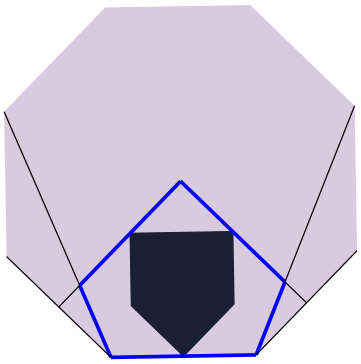
Right – pattern created with three copies of template



The blue part can be constructed from a regular octagon inscribed in a square. In both cases, we use the right angle to draw the pattern. The table below shows each of these polygons and its relation with the regular octagon.



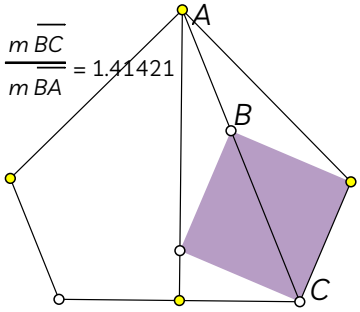




Polygons used in both tesselations

The drawings here show how each of the polygons from Saruhan Avanos is related to the regular octagon. In many octagonal patterns, we often observe the relation d/s where d is a diagonal of a square with side a, and $d/s=\sqrt{2}=1.41421...$ In the right drawing, BC is the diagonal of a square with an edge equal to AB.

There are more shapes that can be used in tessellations for octagonal patterns from adornments in Anatolian Seljuk architecture.



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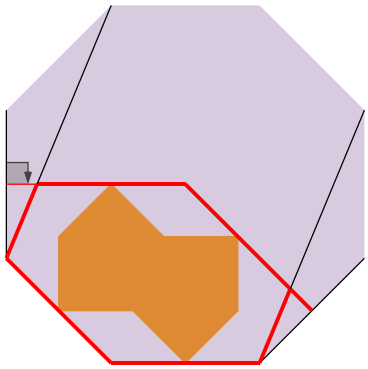
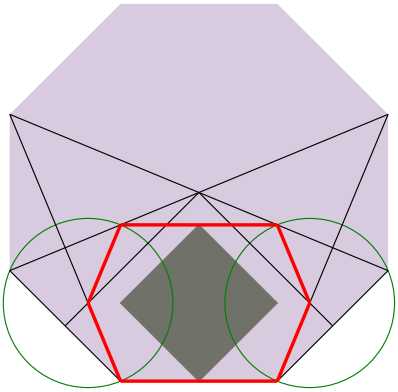
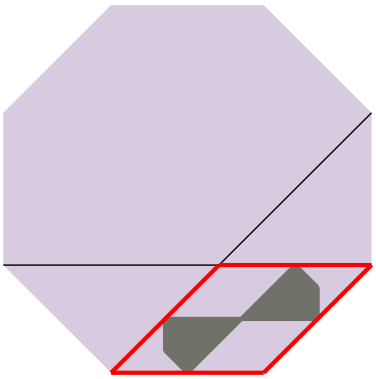
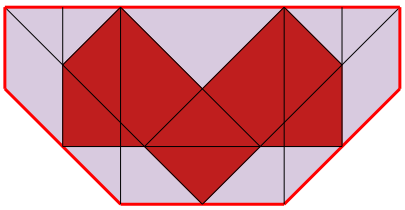
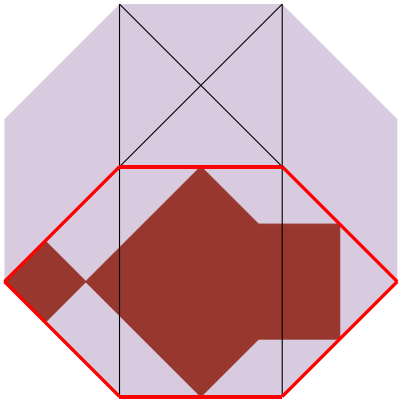
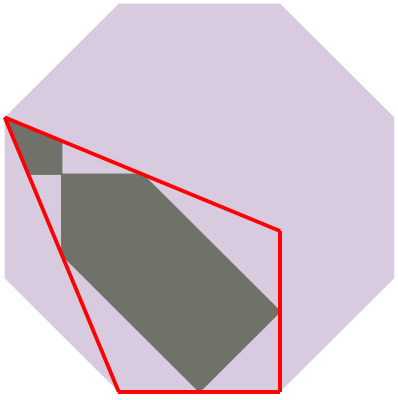
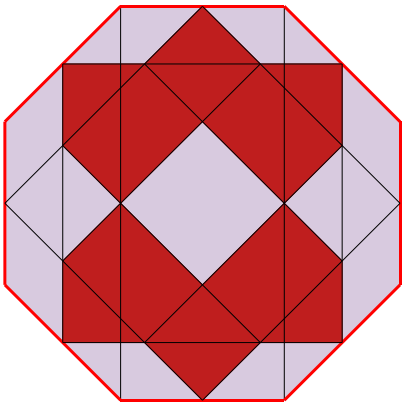
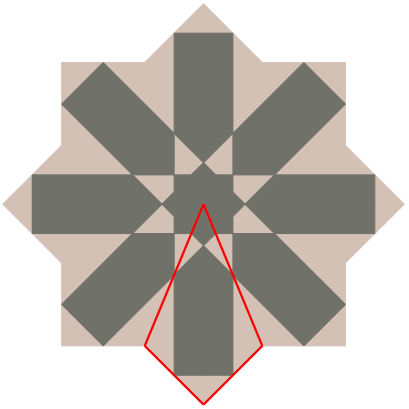
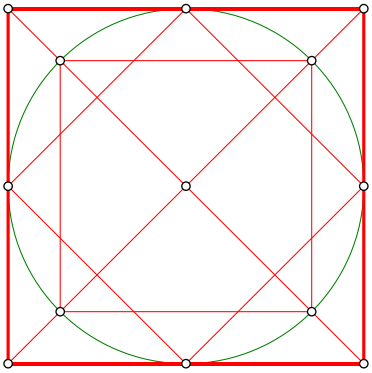
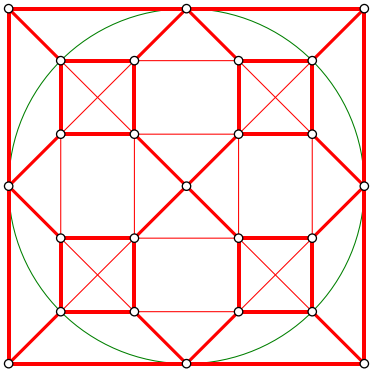
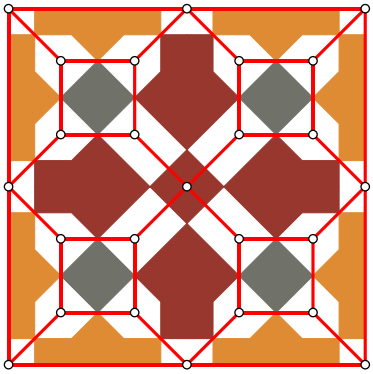
Selected polygons used in octagonal geometric patterns

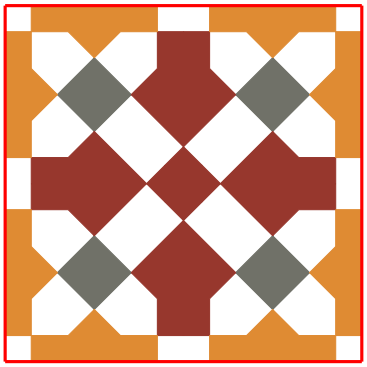








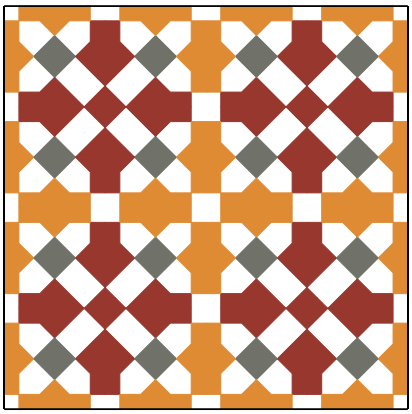
The number of polygons that can be used in patterns based on octagonal geometry is much larger, but here, we show only those frequently seen in Seljuk architecture. There are also variations of motifs drawn in these polygons. We used only 90-degree angles for pattern creation, but one may try other angles.



Below, we show some examples of patterns seen in Anatolian Seljuk architecture.

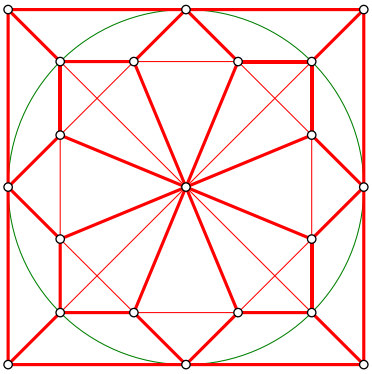


(*)


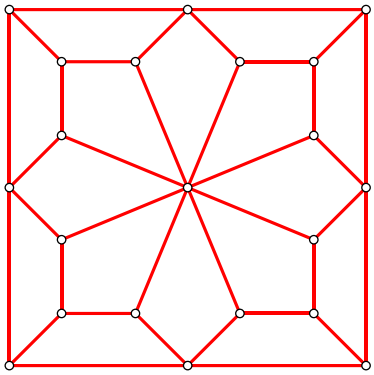


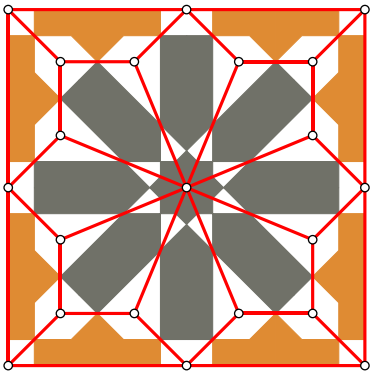


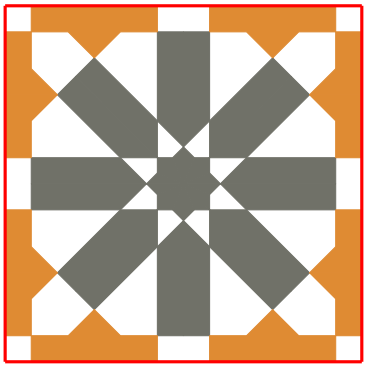
Geometric pattern from Erzurum, Ulu Camii

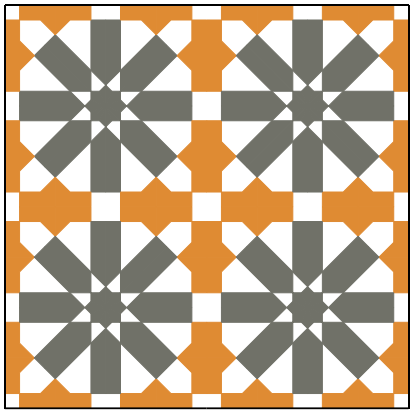
This is quite a popular design. It can be made as an interlaced pattern or kundekari design. An interesting feature of it is the large octagonal star (see drawing (*) on the previous page). The interior of the star can be modified in a few different ways.





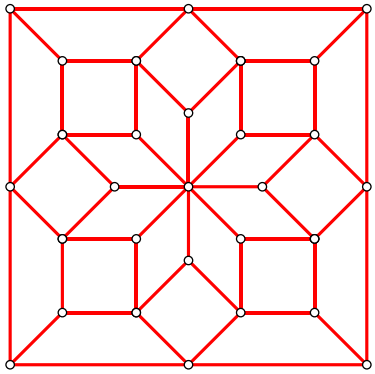


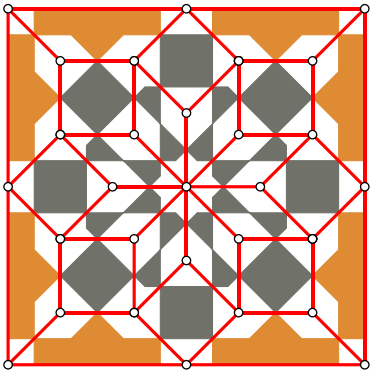


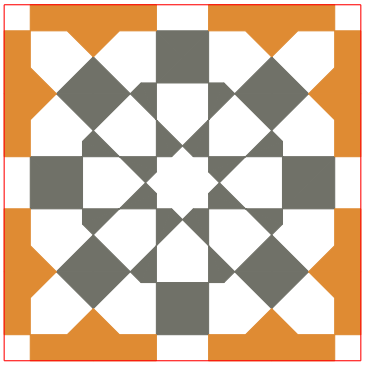


Niğde Alaaddin Mosque pattern

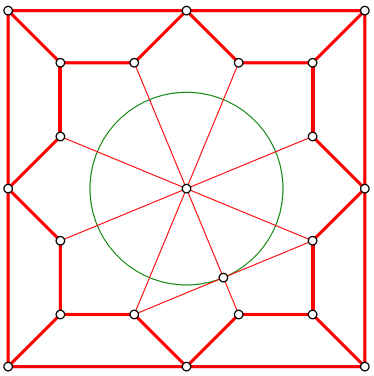
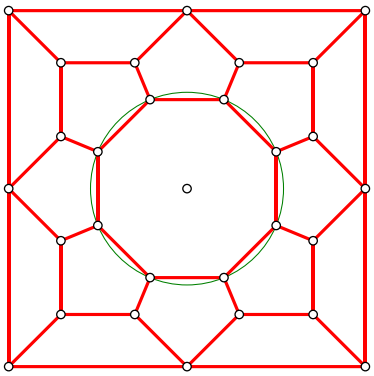
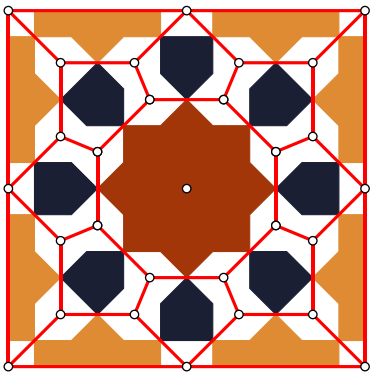
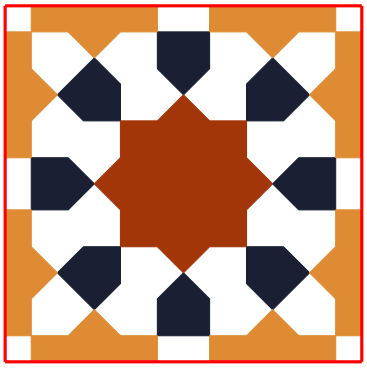
Here is another very popular geometric pattern. The original pattern was carved in stone using interlace feature. The pattern is very easy to design as a 3D kundekari work.





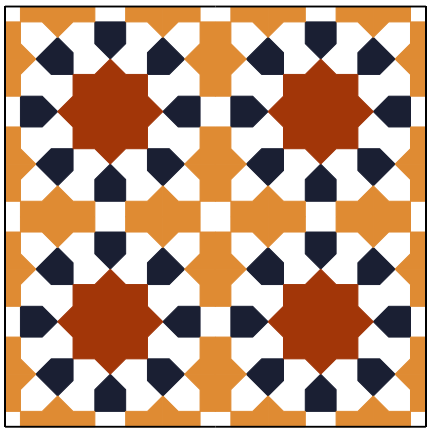


Pattern from Haci Ferruh Mosque in Konya

Pattern from Sultan Hani in Kayseri

It is a very popular design that exists in many places all over Turkey.



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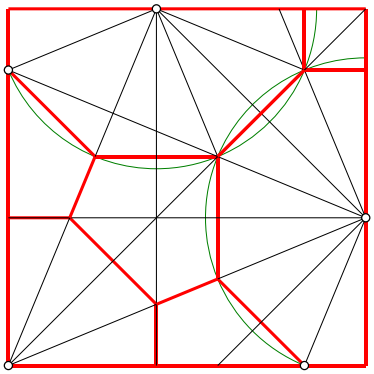
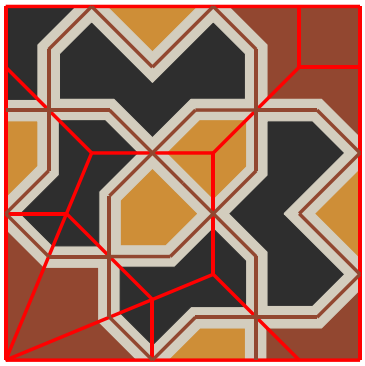
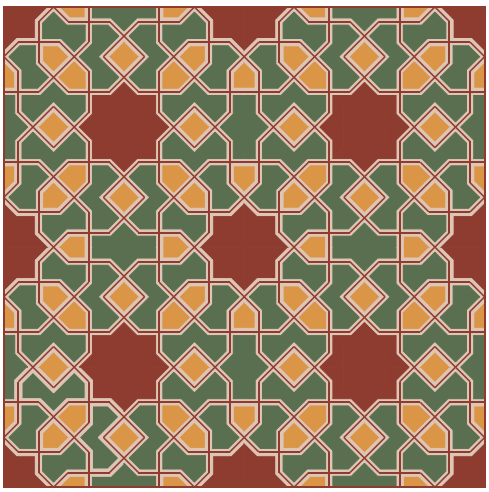




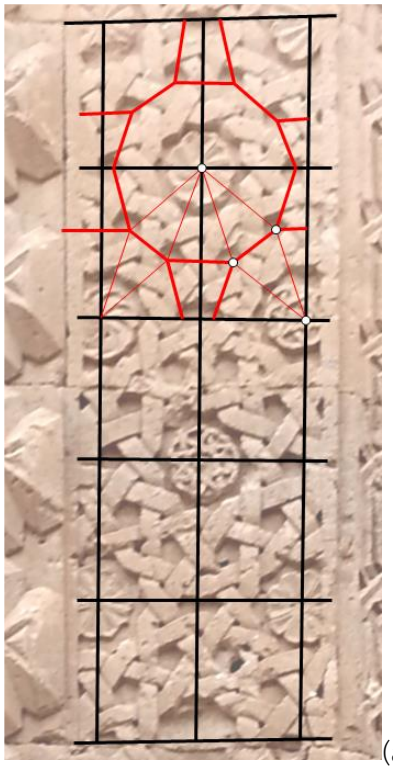
Pattern from Selimiye Mosque in Edirne

This pattern is frequently seen in many places in Turkey.

In Anatolian Seljuk architecture, we can find many interesting geometric concepts. Many of them are very simple and can be used for students' projects in mathematics. Some others may require more experienced students.

From octagonal geometry, we can move to decagonal geometry, where we get completely different angles, polygons, and tessellations. Decagonal geometry was the most frequently used by Anatolian architects and Ottoman kundekari masters. Some adornments on large doors in Ottoman mosques are based on concepts never used, or perhaps not even known, in other regions. This is the topic for another paper or book. Now, let us concentrate on decagonal patterns from Anatolian architecture.

Anatolian Seljuk patterns using decagonal geometry



(a)

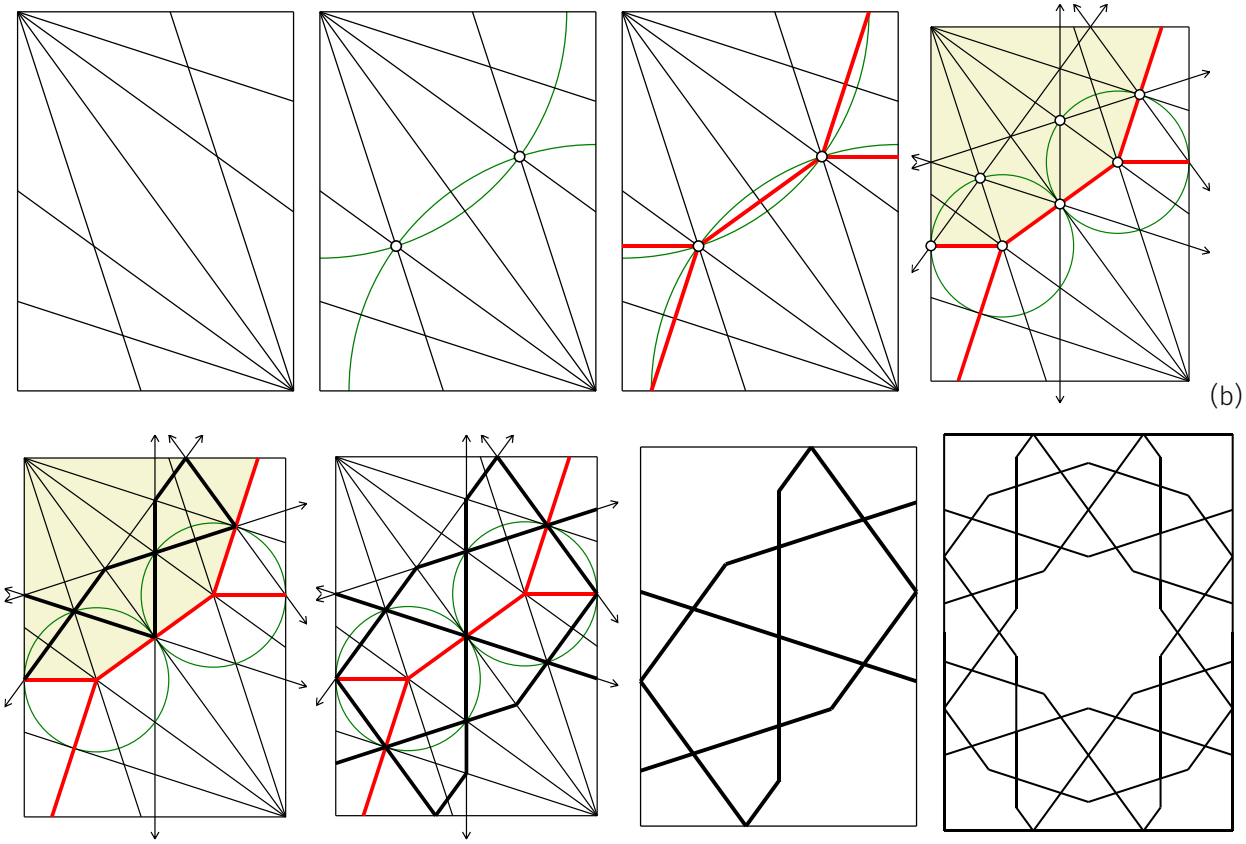
Fragment of a wall in Agzikara Han

The right drawing shows how one can approach this design. Here, We can identify a series of tangent decagons and a pattern with regular pentagons and decagonal stars.

Below are shown all the steps necessary to reconstruct this pattern.

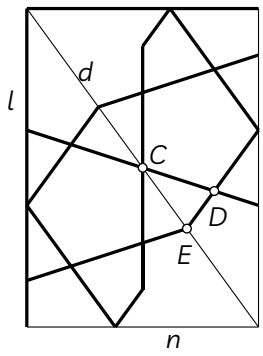
Note how we obtained the regular pentagons (drawing (b)).

The last drawing shows a template for this design. All other rectangles on the illustration (a) are copies of this template.



(b)

The polygons in this construction are standard elements frequently seen in tessellations for decagonal designs. The next table shows selected polygons used in decagonal designs.



$\frac{m\,\overline{DC}}{m\,\overline{DE}} = 1.61803$

$\frac{m\,d}{m\,n} = 1.70130$

$\frac{m\,l}{m\,n} = 1.37638$

$\frac{m\,d}{m\,l} = 1.23607$

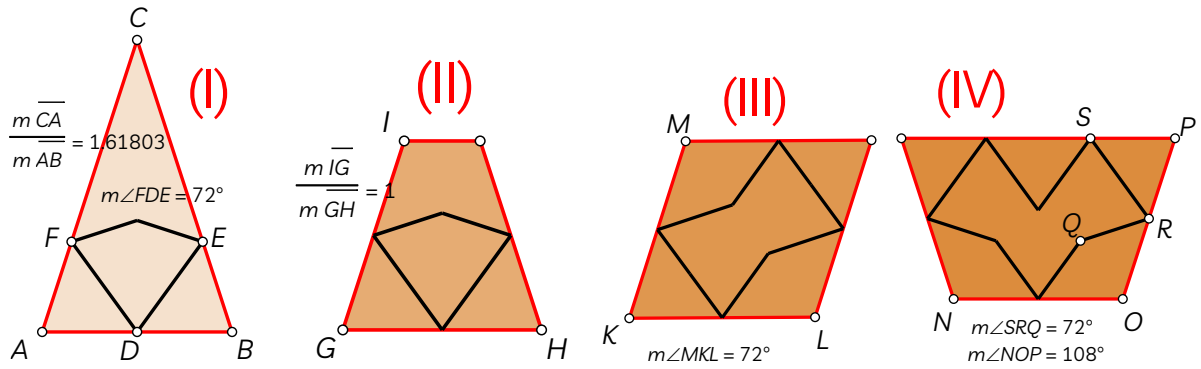
Selected geometric properties of decagonal patterns from Anatolia

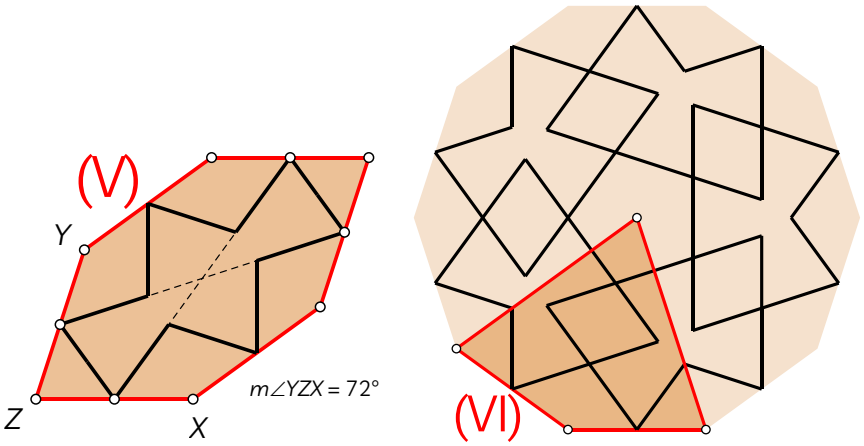
The drawing shows ratios of selected elements of the last design. The first ratio is the ratio that is well-known as the golden ratio. It often occurs when we deal with decagonal designs of any type. Then, we have other important ratios. Each of the other ratios can be derived from the golden ratio.

The rectangle was produced by dividing the right angle into five equal parts.

IMPORTANT – we described here only one of the decagonal patterns system. The one that is frequently used in Anatolian Seljuk architecture. But there exist a few other types of decagonal patterns (see [2] ...[7]).

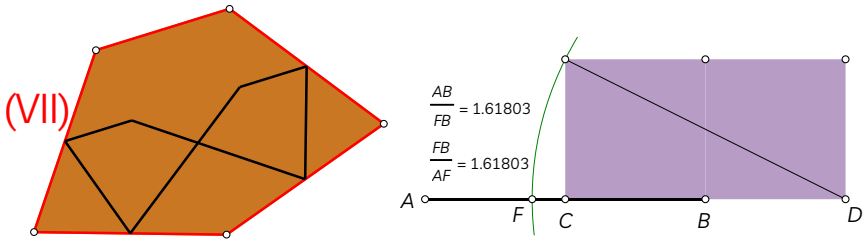
Selected polygons used in decagonal designs from Anatolian Seljuk architecture





Polygons (I) and (II) were used in a pattern from Agzikara. The remaining shapes are often used in other Seljuk adornments.

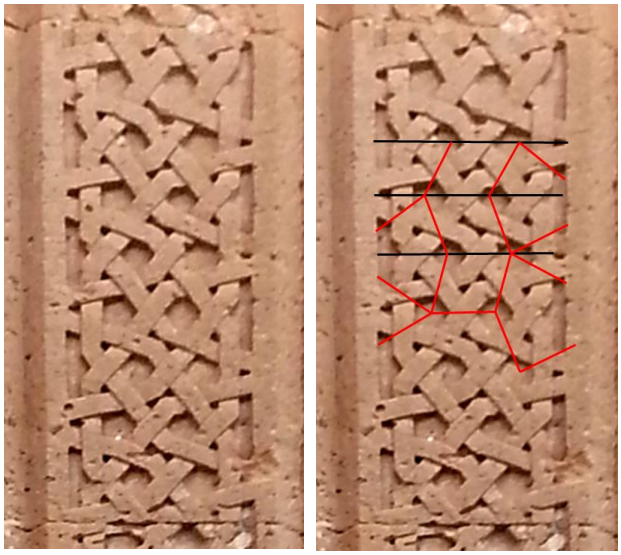
Polygon (VI) can be used as a single tile or combined with four other copies of it. This way, we get a very specific fill for a regular decagon. It is the most frequently used motif in Seljuk design.



In most of these motifs, we often have segments with a ratio equal to the golden ratio.

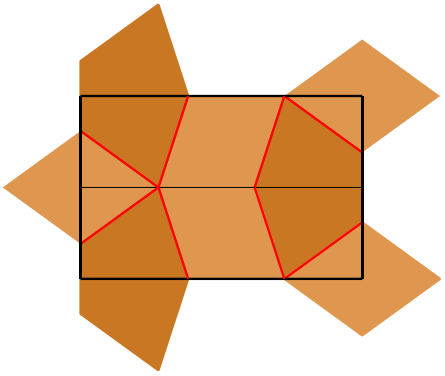
The right drawing shows one of many constructions of the golden section.

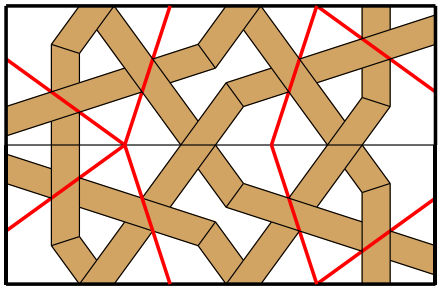
There are a few more decagonal polygons used in Anatolian Seljuk architecture. There is a more detailed description of them in [6] . On the next page, we show examples of Seljuk patterns using these polygons. Usually, these patterns are used as long vertical freezes carved in stone blocks.



Fragment of adornment from Agzikara Han

One can easily notice that the pattern only uses very simple tessellation with two polygons. Its template is a flat strip shown below.





A template for the pattern from Agzikara

To create a long freeze, we must translate this template (no reflections) as often as we need.

In Anatolia, most of such patterns have very wide belts. In the original pattern, the space inside the rhombus is almost closed. In our drawing, we have a small gap.

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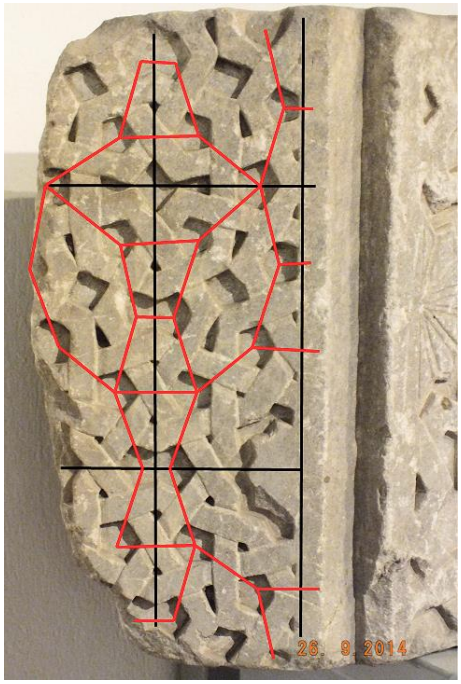
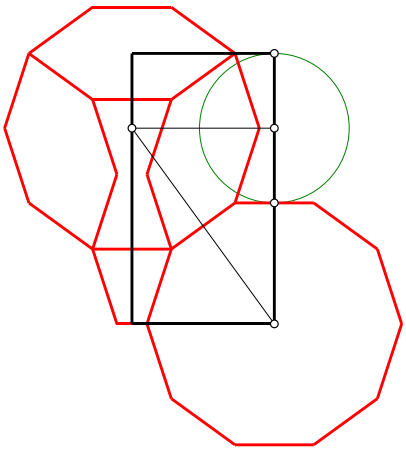


A stone block from Museum in Ince Minaret Madrasah

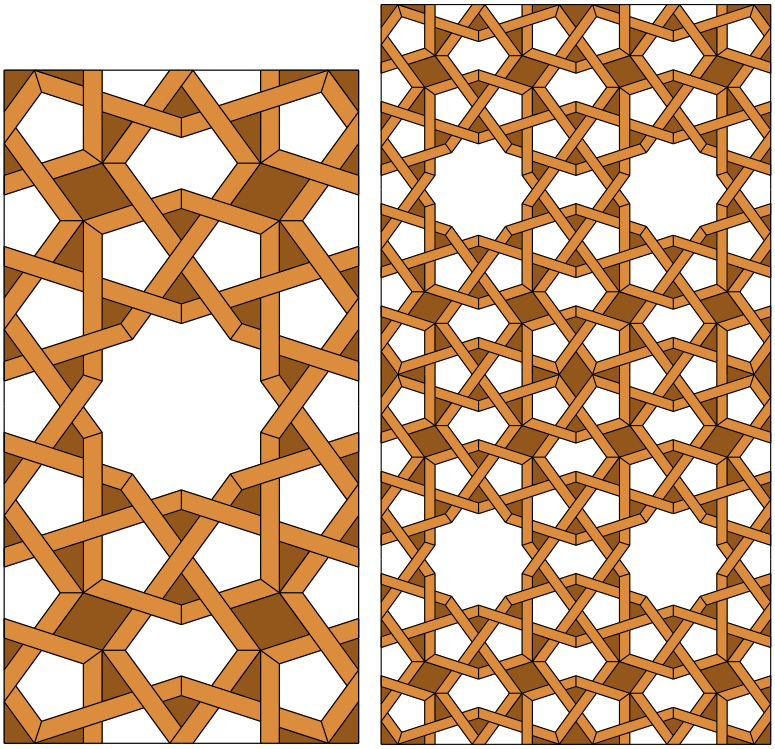
The pattern in this photo is a small fragment of a large adornment. Despite its bad shape, we can reconstruct the whole pattern.

The red drawings show a possible tessellation for this pattern. We need four copies of the rectangle shown in this illustration for an interlace design. For a single-line pattern, we need only one such rectangle.

IMPORTANT – many patterns in Seljuk architecture are clippings of larger patterns. The cut is often done in a wrong place.



Above – construction of the contour for this pattern and its tessellation. The two tangent decagons are the same as in the pattern from Agzikara Han. The top decagon was filled with three hexagons.



Left image – a template for the pattern from Konya

Right image – a larger pattern made from four copies of this template.

In this example, we could fill the regular decagon with a pattern in a few different ways. This way, we can avoid large spaces inside of the decagonal stars.

Decagonal patterns from Anatolia usually are very simple, but we can also find a few unique designs existing only in this part of the world.

Summary

In this paper, we discussed only four types of geometry – the geometry of a square, a regular hexagon, a regular octagon, and a regular decagon. Our discussion was brief, and one can investigate these topics more deeply. For example, we could talk about sequences generated by the golden section or the side and diagonal of square ratio applications. We could talk about triangulations of polygons and complex patterns based on such triangulations. We did not discuss golden polygons in decagonal geometry.

We did not investigate other types of geometries, e.g., the geometry of a regular heptagon, a regular dodecagon, etc. We did not investigate pattern-related topics using two or more geometries, e.g., patterns mixing nonagonal and dodecagonal geometry.

Conclusions

Geometric adornment in Anatolian Seljuk architecture brings us a few things no other school subject can do. It gives us and our students opportunities to investigate:

1. Old architecture and its history, architectural styles, environmental issues, etc.
2. Mathematics of a geometric adornment of historical buildings, i.e., polygons, tessellations, geometric ratios including golden ratio, and various types of symmetries.
3. Technology for geometric design, i.e., Geometer's Sketchpad or GeoGebra.
4. Problem-solving techniques and critical thinking. Some more complex patterns will be an opportunity to learn project management and a top-down design approach.

As one can easily notice, all the subjects mentioned here are the core points of modern education, the so-called STEM concept. On top of this, we are investigating some real objects existing around us in Turkey. There are very few countries with such an advantage.

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[10] https://tarihgezisi.com/kervansaraylar/sarihan-saruhan-kervansarayi-avanos-nevsehir/ (information on Avanos Sari Han).

While writing this article, the most useful resource on Seljuk Anatolian architecture was the catalog by M. Bulut. It contains a huge selection of examples and some important statistical data. The book is available in Turkish only. Thus, the catalog of examples is the most important part for a Western reader.

Many websites are written in Turkish, and one can read information provided there using Google Translate.

All photos in this paper are the sole property of the Author. All drawings were made by the Author using Geometer's Sketchpad. GSP is free software, and it can be downloaded through the web page: https://symmetrica.wordpress.com/gsp-2/